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Application of Fixed point theorem to nonlinear integral equations

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ABSTRACT

Nonlinear integral equations have been a topic of great interest among the mathematicians working in the field of non linear analysis since long time. Krasnoselskii[5] and references given therein. Nonlinearfunctional integral equations have also been discussed in the literature.
e.g. Subrahmanyam and Sundersanam, Ntouyas and Tsamtos and Dhage and Regan etc.

Introduction:

In the present paper, we study a nonlinear functional integral equation of mixed type for the existence result. In particular given a closed and bounded integral J = [0,1] in R, the set of all real numbers, we discuss the following nonlinear functional integral equation (in short FIE)

$$x(t)=q(t)+\int_0^{\mu(t)} k(t,s) f(s,x(\theta(s)))ds + \int_0^{\sigma(t)} v(t,s) g(s,x(\eta(s)))ds \qquad (1.1)$$

Fort J. here $q : J \longrightarrow R$, $k, v : J \times J \longrightarrow R$, $f, g : J \times R \longrightarrow R$ and ,,, $: J \times J \longrightarrow J$.

The FIE (1.1) is general in the sense that it includes the well-known Voltera and Hannerstein integral equations as special cases which have been extensively studied in the literature for various aspects of the solution. The existence for the FIE (1.1) is generally proved by using a fixed point theorem of Krasnoselskii [5], but here in the present paper we obtain the existence result via the following nonlinear alternative recently developed by Dhage and Regan [2]. See also Dhage [1].

Theorem 1.1:

Let B(0,r) and B[0,r] denote respectively the open and closed balls in a Banach space X and let $A, B : X \longrightarrow X$ be two operators satisfying

(a) A is contraction, and
(b) B is completely continuous

Then either

(i) The operator equation Ax + Bx = x has a solution in B[0,r], or
(ii) There exists an element u X with ||u|| = r such that

$$\lambda A\left(\frac{u}{\lambda}\right)+\lambda Bu = u$$

For some λ (0,1).

1.1 Main Result:

Let M(J,R) and B(J,R) respectively denote the spaces of measurable and bounded real-valued functions on J. we shall seek the solution of the FIE (1.1) in the space Bm(J,R) of all bounded and measurable real valued functions on J define a norm ||.|| in BM(J,R) by $\|x\| \max_{t \in J} |x(t)|$

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Clearly BM(J,R) becomes a Banach space with this norm. We need the following definition in the sequel.

Definition 1.1

A mapping : J x R ⟶ R is said to satisfy Caratheodory condition or simply is called Caratheodory if

(i) t ⟶ (t,x) is measurable for each x R.

(ii) x ⟶ (t,x) is continuous almost everywhere for t J, and

(iii) for each real number r > 0, there exists a function
hr L1(J,R)
such that

$$|\beta(t,x)| \le h_r(t), \quad a.e.\ t \in J$$

For all x R with |x| ≤ r.

We consider the following hypothesis in the sequel.

(H0) The functions , , : J ⟶ J are continuous.

(H1) The function q : J ⟶ R is bounded and measurable.

(H2) The functions k, v : J x J ⟶ R are continuous.

(H3) There exists a function α L1(J, R) such that α (t) > 0, a.e. t J and

$$|f(t,x)-f(t,y)| \le \alpha(t)|x-y|, \quad a.e.\ t \in J$$

For all x, y R.

(H4) The function g(t,x) is L1-Caratheodory.

(H4) There exists a non-decreasing function :[0,] ⟶
[0,] and a function L1(J,R) such that (t) > 0, a.e. t J and

$$|g(t,x)| \le \phi(t)\psi(|x|), \quad a.e.\ t \in J$$

For all x R.

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